Integration

Rules of Integration

$f(x)$	a	ax^n	e^{ax}	$\frac{1}{x}$	$\sin ax$	$\cos ax$	Product $(f(x))^n f'(x)$	Quotient $\frac{f'(x)}{f(x)}$
$\int f(x)dx$	$ax+C$	$\frac{ax^{n+1}}{n+1}+C$	$\frac{e^{ax}}{a}+C$	$\ln x+C$	$-\frac{\cos ax}{a}+C$	$\frac{\sin ax}{a}+C$	$\frac{(f(x))^{n+1}}{n+1}+C$	$\ln(f(x))+C$

Examples

1) $\int\left(3x^{\frac{5}{2}}-x^{\frac{3}{2}}+5\right)dx=\frac{6}{7}x^{\frac{7}{2}}-\frac{2}{5}x^{\frac{5}{2}}+5x+C$ 2) $\int\frac{(x-2)(x-3)}{x}dx=\int\left(x-5+\frac{6}{x}\right)dx=\frac{1}{2}x^2-5x+6\ln x+C$

3) $\int(4\sin(2x)+6e^{3x})dx=-2\cos(2x)+2e^{3x}+C$ 4) $\int\sin^2(3x)dx=\int\frac{1}{2}(1-\cos(6x))dx=\frac{1}{2}(x-\frac{1}{6}\sin(6x))+C$

5) $\int(\sin x+1)(\cos x-2)dx=\int(\sin x\cos x-2\sin x+\cos x-2)dx=-\frac{1}{4}\cos(2x)+2\cos x+\sin x-2x+C$

6) $\int x^4(x^5+3)^3dx=\frac{1}{5}\int 5x^4(x^5+3)^3dx=\frac{1}{20}(x^5+3)^4+C$ 7) $\int\frac{3x+1}{3x^2+2x+1}dx=\frac{1}{2}\ln(3x^2+2x+1)+C$

8) $\int_1^4 3\sqrt{x}dx=\int_1^4 3x^{\frac{1}{2}}dx=\left[2x^{\frac{3}{2}}\right]_1^4=\left[2\sqrt{x^3}\right]_1^4=\left(2\sqrt{4^3}\right)-\left(2\sqrt{1^3}\right)=16-2=14$

9) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\left(\frac{x}{3}\right)dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\left(\frac{1}{3}x\right)dx=\left[3\sin\left(\frac{1}{3}x\right)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\left(3\sin(\frac{\pi}{6})\right)-\left(3\sin(-\frac{\pi}{6})\right)=3\left(\frac{1}{2}\right)-3\left(-\frac{1}{2}\right)=3$

Exercises

1) $\int\left(x^2+\sqrt{x}\right)dx$ 2) $\int\frac{x^2(x^2-4)}{x+2}dx$ 3) $\int\frac{3x+2}{\left(3x^2+4x\right)^3}dx$ 4) $\int(\sin x+\cos x)^2dx$

5) $\int_0^{\frac{\pi}{2}}\cos xdx$ 6) $\int_1^5\frac{1}{x}dx$ 7) $\int_{-2}^{-1}(3x^2+2)dx$ 8) $\int_{-2}^4 e^{\frac{1}{2}x}dx$ 9) $\int_0^{\pi}\sin\left(\frac{x}{4}\right)dx$ 10) $\int_3^4\frac{1}{2x}dx$

Answers

1) $\frac{1}{3}(x^3+2\sqrt{x^3})+C$ 2) $\frac{1}{4}x^4-\frac{2}{3}x^3+C$ 3) $-\frac{1}{4(3x^2+4x)^2}+C$ 4) $x-\frac{1}{2}\cos(2x)+C$

5) 1 6) 1.6 7) 9 8) 14.04 9) 1.17 10) 0.144