Power Maths - Version 1

by Sidney Schuman (published in Mathematics Teaching, June 2002)

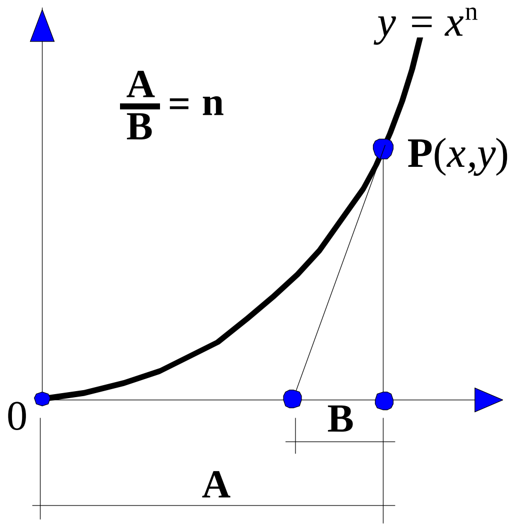

Figure 1

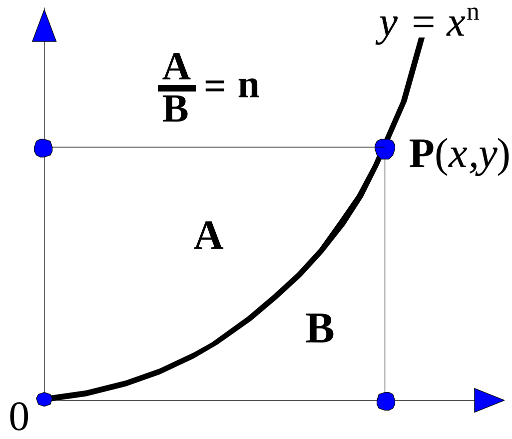

Figure 2

Power Maths is the name given to a graph-based non-rigorous geometric method of teaching the calculus power rules. Students using this method can discover the rules without making any gradient calculations and without first having to understand the concept of the limit. The method is based on the curious property that each calculus power rule can be reduced to a simple geometric ratio. The differential power rule can be reduced to a ratio of dimensions (Figure 1) and the integral power rule can be reduced to a ratio of areas (Figure 2). These ratios are easy for a student to establish by investigation, leaving only some elementary algebra to make the connection from ratio to rule, as described below.

To find the differential power rule, pre-drawn graphs of $y = x^2$, $y = x^3$, $y = x^4$, etc will be needed. (There are several ways of producing accurate graphs, for example using kSoft's *Graphmatica*.) Draw a tangent line at a specific point *P* and measure the dimensions *A* and *B*. Confirm that $A/B = n$ by using several points on each graph and repeating this routine. This can be remembered as "*power = apex over base*". The rule is then derived as follows:

$$\frac{A}{B} = n, \quad but \quad A = x \quad so \quad \frac{x}{B} = n \quad \therefore B = \frac{x}{n}; \text{ gradient of tangent } = \frac{y}{B} = \frac{x^n}{B} = \frac{x^n}{\frac{x}{n}} = nx^{n-1}$$

This is called the **Power Decrease Rule** using notation: $D\left(x^n\right) = nx^{n-1}$

To find the integral power rule, sketch graphs of $y = x^2$, $y = x^3$, $y = x^4$, etc will be needed, drawn for values of *x* between 0 and 10. Using the mid-ordinate rule with ten strips, the area of region B can be calculated approximately as: $0.5^n + 1.5^n + 2.5^n + \cdots + 7.5^n + 8.5^n + 9.5^n$. The rectangle area is *xy* and hence area $A = xy - B$. Confirm that $A/B = n$ with a very small error (which could be made smaller by using more mid-ordinate strips). This can be remembered as '*power = above over below*'. The rule is derived as follows:

$$\frac{A}{B} = n \quad \therefore A = Bn; \quad A + B = xy \quad \therefore A + B = x^{n+1} \quad \therefore Bn + B = x^{n+1} \quad B\left(n+1\right) = x^{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$$

This is called the **Power Increase Rule**, using the notation: $I\left(x^n\right) = \frac{x^{n+1}}{n+1}$

Practice in using each rule on simple monomial functions can be followed by self-checking exercises using both rules, enabling the student to discover their inverse relationship. The student should now be more amenable to those aspects of calculus (the concept of the limit and the formal notation) that cause the most anxiety.