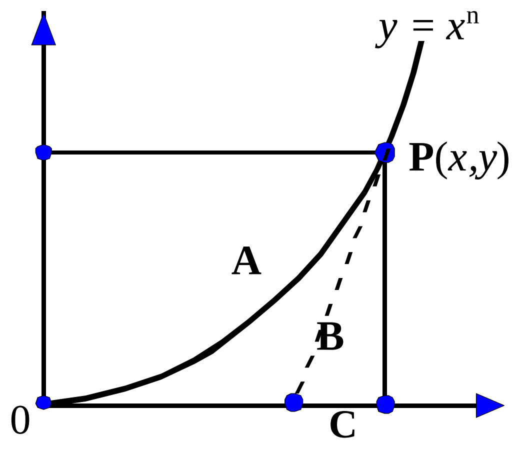
Power Maths - Version 2 by Sidney Schuman (published in Mathematics Teaching, November 2006)

In the diagram below, A and B are the areas of the regions of the rectangle divided by the curve $y = x^n$.



$$x = \frac{A}{B} \times C$$

This curious equation, which is true for all values of *x* and *n*, is the basis for a useful pre-calculus exercise. The student can be prompted to deduce each calculus power rule from it, as in the routine shown below.

How can the area of the rectangle be expressed?

There are two ways: rectangle area = *A* + *B*, and also rectangle area = $xy = x \times x^n = x^{n+1}$.

Putting these together means that $A + B = x^{n+1}$.

From the investigation we notice that the ratio of the areas is always equal to the power of *x*.

So we have $\frac{A}{B} = n$; therefore, $A = Bn$, and we can substitute this into $A + B = x^{n+1}$.

Thus, $Bn + B = x^{n+1} \quad \therefore B(n+1) = x^{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$.

You've probably seen this written formally as $\int x^n dx = \frac{x^{n+1}}{n+1}$, the integral power rule.

What do you know about the gradient of a curve at any point?

It is the same as the gradient of the tangent line at the point *P*(*x*,*y*) on the curve.

But this is the hypotenuse of a right-angled triangle, whose gradient is $\frac{height}{base} = \frac{x^n}{c}$.

Since $\frac{A}{B} = n$, by substitution into the equation $x = \frac{A}{B}c$ we get $x = nc$.

What is the base of the triangle? By simple transposition, the base is $c = \frac{x}{n}$.

So the gradient of the tangent is $\frac{height}{base} = \frac{x^n}{\frac{x}{n}} = x^n \times \frac{n}{x} = nx^{n-1}$.

You've probably seen this written formally as $\frac{dy}{dx} = nx^{n-1}$, the differential power rule.