

Trigonometric Equations

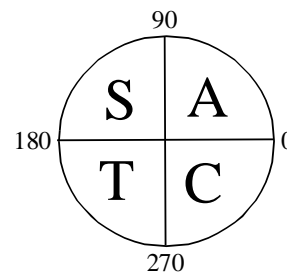
The table gives trigonometric ratios of the common angles in the first quadrant. When solving these equations, the first answer (as given by a calculator) is called the Principal Value.

Secondary Values of these angles occur as shown in the diagram. This shows where angles with **positive** trigonometric ratios occur, those with negative ratios are not shown but their positions can be deduced.

To find Secondary Values from known Principal Values, use:

$$SV = \pi - PV \text{ for } \sin(\theta), \quad SV = -PV \text{ for } \cos(\theta), \quad SV = \pi + PV \text{ for } \tan(\theta).$$

Degrees	0	30	45	60	90
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin (θ)	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos (θ)	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
tan (θ)	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	∞



Example

Solve the equation $\tan(x) = \frac{1}{3}\sqrt{3}$

For $\tan(x) = \frac{1}{3}\sqrt{3}$ the PV is: $\frac{\pi}{6}$ \therefore SV = $\pi + PV = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

For $-\pi < x < \pi$ the SV is: $-\frac{5\pi}{6}$, so the solution is written as: $x = \frac{\pi}{6}, -\frac{5\pi}{6}$

Exercises

Solve the following, giving your answers as fractions of π in the interval $-\pi < x < \pi$

1. $\sin(x) = -0.5$ 2. $\cos(x) = 0.5$ 3. $\tan(x) = -\sqrt{3}$ 4. $\sin(x) = \frac{1}{2}\sqrt{2}$

5. $\cos(x) = -\frac{1}{2}\sqrt{2}$ 6. $2\sin^2(x) - \sin(x) = 0$ 7. $\tan^2(x) + 2\tan(x) + 1 = 0$

Answers 1) $-\frac{6\pi}{5}, -\frac{4\pi}{5}$ 2) $\frac{3\pi}{2}, \frac{5\pi}{2}$ 3) $-\frac{3\pi}{4}, \frac{3\pi}{4}$ 4) $\frac{3\pi}{4}, \frac{5\pi}{4}$ 5) $\frac{4\pi}{3}, \frac{2\pi}{3}$ 6) $0, \frac{6\pi}{5}, \frac{9\pi}{5}$ 7) $-\frac{4\pi}{3}, \frac{4\pi}{3}$