Normal Distribution
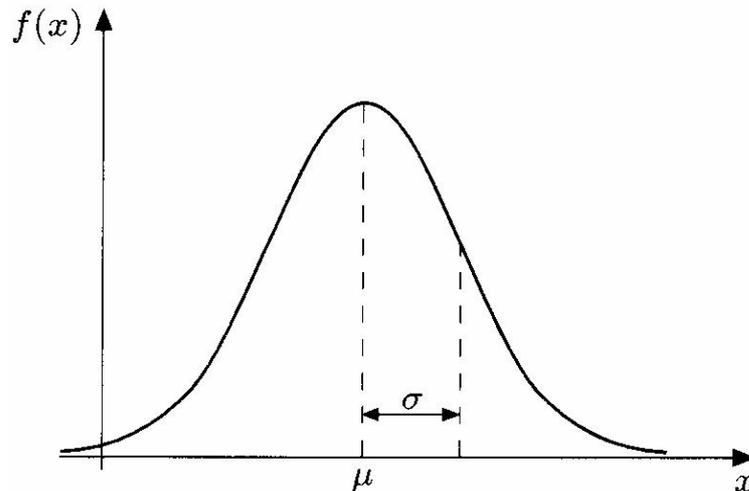
Variation of data can, where feasible, be modelled by the Normal Distribution Curve, in which the mean value of the data (μ) is shown by a vertical line at the centre of the curve. The spread of the data is determined by the value of its standard deviation (σ) - the higher the value the flatter the shape of the curve. Here is a typical curve.

The area under a typical normal distribution curve is equal to 1 and the proportion of area between any two values of x can be obtained using statistic 'Normal Tables'. The relevant statistic outputs are given in the following exercises in the form 'specified value ⇨ proportion of area to the left'.

1 A normal curve with mean 15 and standard deviation 3 has the following outputs: 12⇨0.159, 17⇨0.748. What are the proportions (a) more than 12 (b) more than 17 (c) between 12 and 17 (d) between 15 and 17 ?

2 A normal curve with mean 8 and standard deviation 2.3 has the following outputs: 6⇨0.192, 11⇨0.520. What are the proportions (a) more than 6 (b) more than 11 (c) between 6 and 11 (d) between 8 and 11 ?

3 A manufacturer of washing-up liquid produces a liquid with a new fragrance, in bottles labelled to indicate a volume of 250ml. The variation in the actual volume of the washing-up liquid in these bottles can be modelled by a normal distribution with mean 250.8ml and standard deviation 4.3ml. Find (a) the value above which approximately 80% of the volumes lie (b) a range of values, centred on the mean, within which approximately 90% of the volumes lie.   [0.2⇨247.2, 0.05⇨243.7, 0.95⇨257.9]

4 Variation in the time for a visit to a particular website can be modelled by a normal distribution with mean 32.5 minutes and standard deviation 7.5 minutes. (a) What is the time within which 50% of visits are completed? (b) What is the time within which 90% of visits are completed? (c) Find a range of values for the time, centred on the mean, within which approximately 99% of visits are completed.
                    [0.5⇨32.5, 0.9⇨42.1, 0.005⇨13.2, 0.995⇨51.8]

5 Values of the variant in a normal distribution are often given in the form $\mu \pm k\sigma$. State the proportion of values which are less than k standard deviations from the mean when k = (a) 1 (b) 1.5 (c) 2.5 (d) 3.
                    [-1 ⇨ 0.159, -1.5 ⇨ 0.134, -2.5 ⇨ 0.012, -3 ⇨ 0.002]

6 The proportion of area under a normal distribution curve within a given number of standard deviations of the mean (ie between the vertical lines $x = \mu - k\sigma$ and $x = \mu + k\sigma$) holds true irrespective of the values of the mean and standard deviation. What are those for k = 1, k = 2 and k = 3 ?   [Answers not given]