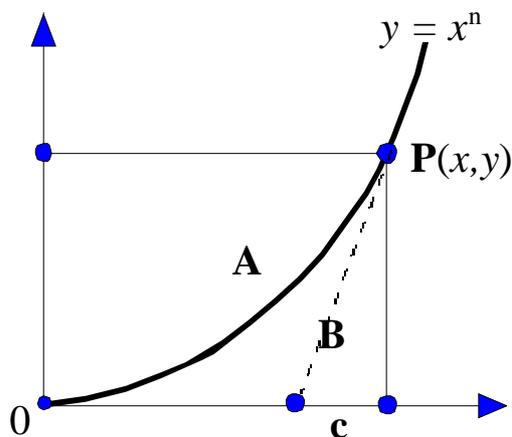


## Another look at the calculus power rules

by Sidney Schuman (Published in The Mathematical Gazette, July 2005)



At point  $P$  on the graph of  $y = x^n$  ( $x > 0$ ), a rectangle is formed by lines perpendicular to the coordinate axes and a right triangle is formed by a line tangent to the curve. The rectangle is divided by the curve into regions  $A$  and  $B$ , the triangle has base  $c$ . A previous article of mine (Tinkering with the calculus power rules) used two diagrams to support the algebra that showed how, on the one hand,  $A/B = n$  is obtained from the integral power rule and, on the other hand,  $x/c = n$  is obtained from the differential power rule. These were treated separately with no attempt to show a connection between them.

This connection is, of course, a heavily-disguised version of the inverse relationship between the calculus power rules. This relationship is shown easily enough with a standard piece of algebra, but this begs an interesting, albeit naive, question. 'Why is an area rule the inverse of a gradient rule?' It occurred to me that combining these diagrams into a single diagram (above) might provide a way of addressing that question.

This seems to suggest that, as point  $P$  moves along the curve, the tangent line 'projects' the ratio  $A/B$  onto the  $x$ -axis. In other words, just as region  $B$  is always  $1/n$  th of region  $A$ , so the base of the triangle is always  $1/n$  th of the base of the rectangle. Since these ratios are merely 'boiled-down' versions of the calculus power rules, this could be a way of showing graphically the reason why they are connected.

I wonder if it is possible to devise a mathematical formulation, based on this 'dynamic' geometry, that answers my naive question simply and convincingly?