

Calculus Revision

Standard derivatives:

$$f'(c) = 0, f'(x^n) = nx^{n-1}, f'(\sin(ax)) = a \cos(ax), f'(\cos(ax)) = -a \sin(ax), f'(\tan(ax)) = a \sec^2(ax)$$
$$f'(e^{ax}) = ae^{ax}, f'(\ln(ax)) = \frac{1}{x}, f'(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, f'(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, f'(\tan^{-1} x) = \frac{1}{1+x^2}$$

Product rule:

If $f(x) = g(x)h(x)$ then $f'(x) = f(x)g'(x) + f'(x)g(x) \dots$ or ... if $y = uv$ then $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

Quotient rule:

If $f(x) = \frac{g(x)}{h(x)}$ then $f'(x) = \frac{h(x)g'(x) - h'(x)g(x)}{(h(x))^2} \dots$ or ... if $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{1}{v^2} \left(v \frac{du}{dx} - u \frac{dv}{dx} \right)$

Composite rule:

If $f(x) = g(h(x))$ then $f'(x) = g'(h(x))h'(x) \dots$ or ... if $u = f(x)$ and $y = g(u)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Standard integrals (constant of integration not shown):

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \int \frac{1}{x} dx = \ln x, \int e^{ax} dx = \frac{1}{a} e^{ax}, \int \cos(ax) dx = \frac{1}{a} \sin(ax), \int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$
$$\int \sec^2 x dx = \frac{1}{a} \tan(ax), \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x, \int -\frac{1}{\sqrt{1-x^2}} dx = \cos^{-1} x, \int \frac{1}{1+x^2} dx = \tan^{-1} x$$

Product Rule: $\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1}$ Quotient Rule: $\int \frac{f'(x)}{f(x)} dx = \ln(f(x))$

Integration by substitution: $\int f(g(x))g'(x)dx = \int f(u)du$ where $u = g(x)$

Integration by parts: $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

Differentiate:

1. $f(x) = \sqrt{\frac{x^3}{2} - \frac{2}{x^3}}$ 2. $f(x) = \frac{e^{7x}}{x^2}$ 3. $f(x) = 3x^2 \ln x$ 4. $f(x) = \ln(6x) \cos(2x)$ *

5. $f(x) = \frac{\ln(x-1)}{\cos x}$ 6. $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$ 7. $f(x) = \sin^{-1}(\sqrt{x})$ 8. $f(x) = \sin^{-1}\left(\frac{1}{\sqrt{x}}\right)$

Integrate:

9. $f(x) = x^5 e^{(1+x^6)}$ 10. $f(x) = x^3 \cos(1+x^4)$ 11. $f(x) = \frac{\sin x}{2 + \cos x}$ 12. $f(x) = \frac{\ln x}{x}$

13. $f(x) = \sec^2(4x)$ 14. $f(x) = x e^{\frac{1}{4}x}$ 15. $f(x) = x^2 \ln(3x)$ 16. $f(x) = x \cos(4x)$

17. $f(x) = \frac{x}{1+x^4}$, between 0 and 1 18. $f(x) = \frac{\cos x}{\sqrt{1 - \frac{1}{9} \sin^2 x}}$, between 0 and $\frac{\pi}{2}$

19. $f(x) = x \sin\left(\frac{1}{2}x\right)$, between 0 and $\frac{\pi}{3}$ 20. $f(x) = \sqrt{x^3} \ln x$, between 1 and 4

* Find the second differential for Question 4