Coordinate Geometry

This worksheet covers equations of circles and tangents

Example

Find the equation of the tangent at (5, 1) to the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ and the acute angle between this tangent and the *x*-axis. Sketch the circle showing coordinates of any intercepts and write down its parametric equations.

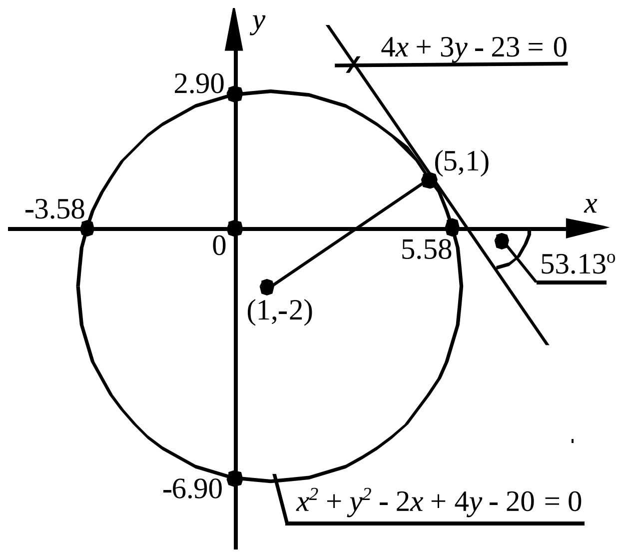
$x^2 + y^2 - 2x + 4y - 20 = 0 \quad \therefore (x-1)^2 + (y+2)^2 = 25$ so the centre is (1, -2), radius 5.

Gradient of radius line through (5, 1) and (1, -2) is: $\frac{1-(-2)}{5-1} = \frac{3}{4}$ and so the gradient of the tangent at (5, 1) is: $-1 \div \frac{3}{4} = -\frac{4}{3}$ (because tangent and radius line are perpendicular).
Using the form: $y - y_1 = m(x - x_1)$ we have: $y - 1 = -\frac{4}{3}(x-5) \quad \therefore 4x + 3y - 23 = 0$

Given that the tangent gradient is: $-\frac{4}{3}$, the acute angle to the *x*-axis is: $\tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$

For intercepts on the *x*-axis, *y* = 0:
$x^2 - 2x - 20 = 0 \therefore x = -3.58 \textbf{ or } 5.58$
For intercepts on the *y*-axis, *x* = 0:
$y^2 + 4y - 20 = 0 \therefore y = -6.90 \textbf{ or } 2.90$
Thus the circle cuts the coordinate axes at:
(-3.58, 0), (5.58, 0), (0, -6.90) and (0, 2.90).

The sketch of the circle shows the equations of the circle and the tangent, plus all other relevant information, given or found.



The parametric equations of the circle are obtained from the 'completed square' form: $(x-1)^2 + (y+2)^2 = 25$. They can be written straight down as: $x = 1 + 5\cos\theta, \quad y = -2 + 5\sin\theta.$

Exercises

Repeat the above calculation for the following circles and points:

1 $\quad x^2 + y^2 + 2x + 4y = 0$, (0, 0)
2 $\quad x^2 + y^2 - 8y + 3 = 0$, (-2, 7)
3 $\quad x^2 + y^2 - 10x - 22y + 129 = 0$, (6, 7)

Answers
1) $x + 2y = 0$, 26.57°, (-2,0),(0,0),(0,-4), $x = -1 + \sqrt{5}\cos\theta$, $y = -2 + \sqrt{5}\sin\theta$
2) $2x - 3y + 25 = 0$, 33.69°, (0,0.39),(0,7.60), $x = \sqrt{13}\cos\theta$, $y = 4 + \sqrt{13}\sin\theta$
3) $x - 4y + 22 = 0$, 14.04°, (no intercepts), $x = 5 + \sqrt{17}\cos\theta$, $y = 11 + \sqrt{17}\sin\theta$