Power Maths Calculations

Integral power rule (sample calculations for x =10)

n = 2 Area B = $0.5^2 + 1.5^2 + 2.5^2 + 3.5^2 + 4.5^2 + 5.5^2 + 6.5^2 + 7.5^2 + 8.5^2 + 9.5^2$ = 332.5
Rectangle area = 1,000 Area A = 1,000 – 332.5 = 667.5 A/B = **2.008**

n = 3 Area B = $0.5^3 + 1.5^3 + 2.5^3 + 3.5^3 + 4.5^3 + 5.5^3 + 6.5^3 + 7.5^3 + 8.5^3 + 9.5^3$ = 2,487.5
Rectangle area = 10,000 Area A = 10,000 – 2,487.5 = 7512.5 A/B = **3.020**

n = 4 Area B = $0.5^4 + 1.5^4 + 2.5^4 + 3.5^4 + 4.5^4 + 5.5^4 + 6.5^4 + 7.5^4 + 8.5^4 + 9.5^4$ = 19,833.625
Rectangle area = 100,000 Area A = 100,000 – 19,833.625 = 80,166.375 A/B = **4.042**

n = 5 Area B = $0.5^5 + 1.5^5 + 2.5^5 + 3.5^5 + 4.5^5 + 5.5^5 + 6.5^5 + 7.5^5 + 8.5^5 + 9.5^5$ = 164,590.625
Rectangle area = 1,000,000 Area A = 1,000,000 – 164,590.625 = 835,409.375 A/B = **5.075**

Results confirm that A/B = n, taking into account the approximate calculation of Area B.

Given that $\frac{\mathrm{A}}{\mathrm{B}} = \mathrm{n}$ (Equation 1) then A = Bn

Given that A + B = $x^{\mathrm{n}+1}$ (Equation 2), substituting A = Bn we get:

Bn + B = $x^{\mathrm{n}+1}$ $\therefore \mathrm{B}(\mathrm{n}+1) = x^{\mathrm{n}+1}$ $\therefore \mathrm{B} = \frac{x^{\mathrm{n}+1}}{\mathrm{n}+1}$, the integral power rule.

Differential power rule

From the pre-drawn graphs of $y = x^2, y = x^3, y = x^4$ and $y = x^5$, results confirm that A/B = n, taking into account the approximate nature of drawing tangents.

Given that $\frac{dy}{dx} = \frac{x^{\mathrm{n}}}{\mathrm{B}}$ (Equation 3) and $\frac{x}{\mathrm{B}} = \mathrm{n}$ (Equation 4)

from Equation 4 we get: $\mathrm{B} = \frac{x}{\mathrm{n}}$ and substituting this into Equation 3 we get:

$\frac{dy}{dx} = \frac{x^{\mathrm{n}}}{\frac{x}{\mathrm{n}}}$ $\therefore \frac{dy}{dx} = \mathrm{n}\, x^{\mathrm{n}-1}$, the differential power rule.