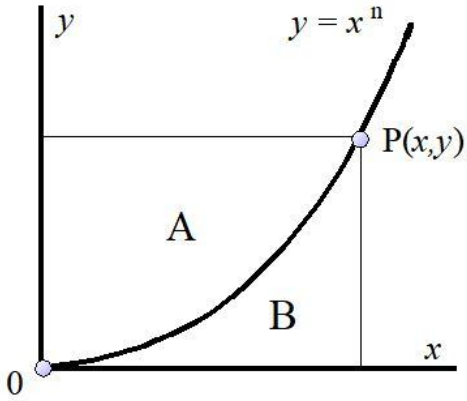
Power Maths Worksheet by Sidney Schuman

Go straight to the integral power rule



On the graph of $y = x^n$ a rectangle has been drawn from point P(*x*,*y*) on the curve. The rectangle is divided by the curve into two regions.

It can be shown using simple algebra that $\frac{A}{B} = n$ *

To confirm this numerically, use an example where *x* = 10 and let the rectangle be divided into 10 vertical strips of 1 unit width.
Calculate area B for n = 2, 3, 4 and 5 using the mid-ordinate rule thus:

$$B = 0.5^n + 1.5^n + 2.5^n + 3.5^n + 4.5^n + 5.5^n + 6.5^n + 7.5^n + 8.5^n + 9.5^n$$

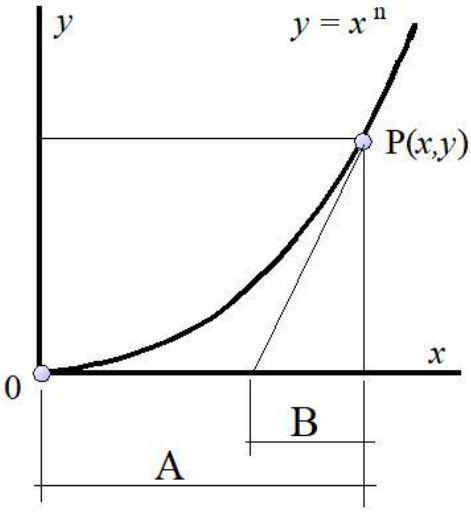
Calculate rectangle area for each value of n: rectangle area = width $\times$ height = $10 \times 10^n = 10^{n+1}$.
Calculate area A for each value of n by subtracting area B from the corresponding rectangle area.

Find the ratio A/B for each value of n. Your calculations should indicate that: $\frac{A}{B} = n$ **Equation 1**

Given that rectangle area = $xy = x \times x^n = x^{n+1}$ and is also A + B, we have $A + B = x^{n+1}$ **Equation 2**

Combine equations 1 and 2 to deduce the integral power rule.

Go straight to the differential power rule



On the graph of $y = x^n$ another line has been drawn tangent to the curve at point P(*x*,*y*). This forms a right triangle with its base on the *x*-axis..

It can be shown using simple algebra that $\frac{A}{B} = n$ *

To confirm this practically, you will need to work on pre-drawn graphs of $y = x^n$ for various values of 'n'. Graphs are available for n = 2, 3, 4 and 5. Download and print these graphs using the link on the home page.

Draw a tangent and an ordinate from a point approximately midway on ech graph.

Check dimensions A and B and confirm that the ratio $\frac{A}{B} = n$.

At P(*x*,*y*) the gradient of the curve $\left(\frac{dy}{dx}\right)$ is equal to the gradient of the tangent,

and in the right triangle, the gradient of the tangent is: $m = \frac{\text{height}}{\text{base}} = \frac{y}{B}$.

So $\frac{dy}{dx} = \frac{y}{B}$ and $y = x^n$ at all points on the curve so we have: $\frac{dy}{dx} = \frac{x^n}{B}$. **Equation 3**

Now, given that $\frac{A}{B} = n$ and noting that A = *x*, we have: $\frac{x}{B} = n$. **Equation 4**

Combine equations 3 and 4 to deduce the differential power rule

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* See 'Tinkering with the calculus power rules' using the link on the home page