The Roots of a Quadratic Equation

The general quadratic equation $ax^2+bx+c=0$ is solved by the formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

There are three possible types of solution, depending on the values of the coefficients *a*, *b*, *c*.

These can be easily identified without having to solve the equation by using the discriminant.

If $\sqrt{b^2-4ac}$ > 0, then it follows that the roots will be real and distinct
If $\sqrt{b^2-4ac}$ = 0, then it follows that the roots will be real and repeated
If $\sqrt{b^2-4ac}$ < 0, then it follows that the roots will be imaginary

Let the roots of a quadratic equation be called α and β

It can be shown that the sum of roots: $\alpha+\beta=-\dfrac{b}{a}$ and the product of roots: $\alpha\beta=\dfrac{c}{a}$

Example

For the quadratic equation $x^2+5x-7=0$, $\alpha+\beta$ = -5 and $\alpha\beta$ = -7

The reciprocals of α,β can be used to form another quadratic equation:

Sum of roots = $\dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{\alpha+\beta}{\alpha\beta}=\dfrac{5}{7}$; product of roots = $\dfrac{1}{\alpha}\times\dfrac{1}{\beta}=\dfrac{1}{\alpha\beta}=-\dfrac{1}{7}$

Thus we have: $\dfrac{b}{a}=-\dfrac{5}{7}$ and $\dfrac{c}{a}=-\dfrac{1}{7}$ which gives us the equation: $7x^2-5x-1=0$

(Use the formula to check that this has roots which are the reciprocal of the roots of $x^2+5x-7=0$)

Alternatively, the squares of α,β can be used to form another quadratic equation:

We use here the identity $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$

Sum of roots = $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=25+14=39$; product of roots = $\alpha^2\times\beta^2=(\alpha\beta)^2=49$

Thus we have $\dfrac{b}{a}=-39$ and $\dfrac{c}{a}=49$; this gives us the equation: $x^2-39x+49=0$

(Use the formula to check that this has roots which are the square of the roots of $x^2+5x-7=0$)

Exercises

Find the quadratic equations whose roots are (a) reciprocals (b) squares of the roots of:

1 $3x^2+5x-2=0$

2 $5x^2-9x+1=0$

Answers: 1(a) $2x^2-5x-3=0$ 1(b) $9x^2-37x+4=0$ 2(a) $x^2-9x+5=0$ 2(b) $25x^2-71x+1=0$