89.35 Another look at the calculus power rules

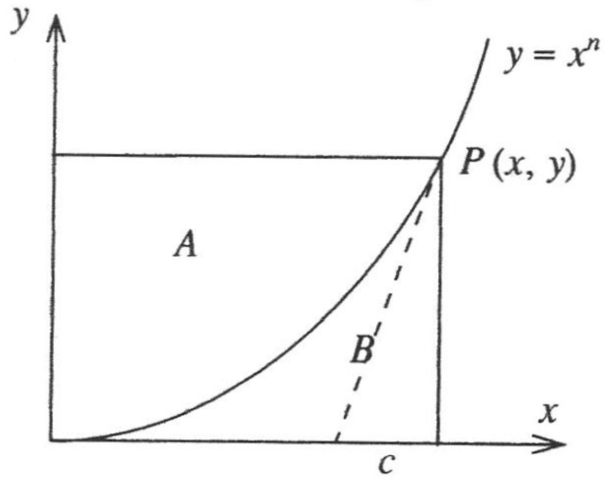

FIGURE 1

At point P on the graph of $y = x^n$ $(x > 0)$, a rectangle is formed by lines perpendicular to the coordinate axes and a right triangle is formed by a line tangent to the curve. The rectangle is divided by the curve into regions A and B, the triangle has base c. A previous article of mine [1] used two diagrams to support the algebra that showed how, on the one hand, $A/B = n$ is obtained from the integral power rule and, on the other hand, $x/c = n$ is obtained from the differential power rule. These were treated separately with no attempt to show a connection between them.

This connection is, of course, a heavily-disguised version of the inverse relationship between the calculus power rules. This relationship is shown easily enough with a standard piece of algebra, but this begs an interesting, albeit naïve, question: '*Why* is an area rule the inverse of a gradient rule?' It occurred to me that combining these diagrams into a single diagram (see Figure 1) might provide a way of addressing that question, but I needed something other than a static diagram. I needed a dynamic picture, which I obtained by using the Java applet at [2].

The applet seems to suggest that, as P moves along the curve, the tangent line 'projects' the ratio A/B onto the x-axis. In other words, just as region B is always $1/n$th of region A, so the base of the triangle is always $1/n$th of the base of the rectangle. Since these ratios are simply 'boiled-down' versions of the calculus power rules, this could be a way of showing graphically the reason why these rules are connected. I wonder, if it is possible to devise a mathematical formulation, based on this 'dynamic' geometry, that answers my naïve question simply and convincingly?

Reference

1. Sidney Schuman, Tinkering with the calculus power rules, *Math. Gaz.* **87** (July 2003) pp. 307-308.
2. www.powermaths.org.uk

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