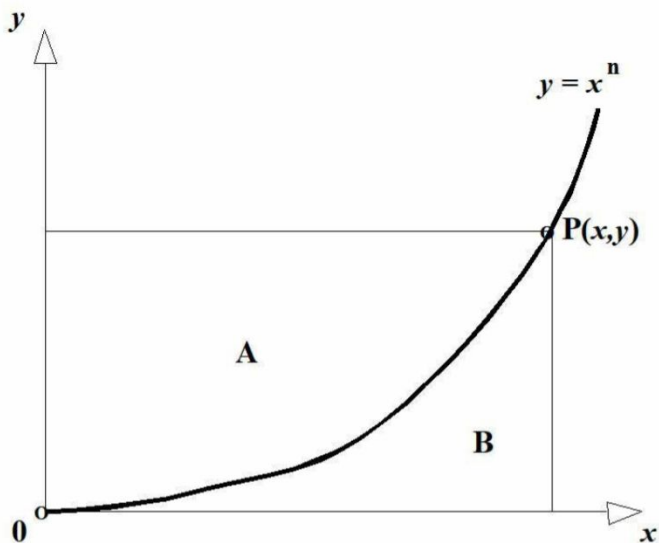


About Power Maths by Sidney Schuman

Power Maths is the name given to a graph-based method of introducing the calculus power rules. Students using this method can discover the rules without having to understand the idea of infinitesimals or the concept of the limit. By investigation they establish the simple geometric ratio $A / B = n$ (see below) and can then use simple algebra to deduce both calculus power rules.

Integral Power Rule



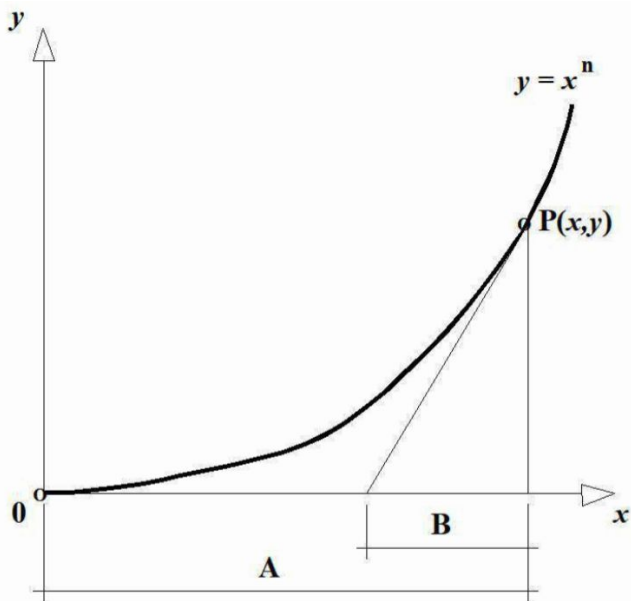
The area of region B can be calculated for $y = x^2$, $0 < x < 10$ using the mid-ordinate rule with ten strips as shown here:
 $0.5^2 + 1.5^2 + 2.5^2 + 3.5^2 + 4.5^2 + 5.5^2 + 6.5^2 + 7.5^2 + 8.5^2 + 9.5^2$.

From this the area of region A is calculated using $A = xy - B$ and then the ratio $A / B = 2$ is found to be true with a very small error. Similar calculations for $y = x^3$, $y = x^4$ & $y = x^5$ should be enough to show that $A / B = n$. The rule can now be deduced as follows:

$$\frac{A}{B} = n \quad \therefore A = Bn; \quad A + B = xy \quad \therefore A + B = x^{n+1}$$

$$\therefore Bn + B = x^{n+1} \quad \therefore B(n+1) = x^{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$$

Differential Power Rule



Pre-drawn graphs of $y = x^2$, $y = x^3$, $y = x^4$ & $y = x^5$ required.

Draw a tangent and an ordinate from a point on the graph of $y = x^2$ and confirm that $A / B = 2$, allowing for drawing error. Repeat for two other points on the graph to confirm.

Use this procedure on the graphs of $y = x^3$, $y = x^4$ & $y = x^5$ and confirm that $A / B = n$ is always true. The rule can now be deduced as follows:

$$\frac{A}{B} = n \quad \therefore \frac{x}{B} = n \quad \therefore B = \frac{x}{n}; \quad \text{at P the curve and tangent have}$$

the same gradient: $m = \frac{y}{B} \quad \therefore m = \frac{x^n}{B} \quad \therefore m = \frac{x^n}{\frac{x}{n}} \quad \therefore m = n x^{n-1}$.

Practice in using each rule on simple monomial functions can be followed by self-checking exercises using both rules, enabling the student to discover their inverse relationship. The student should now be more amenable to those aspects of calculus (infinitesimals, concept of the limit, formal notation) that cause the most anxiety.