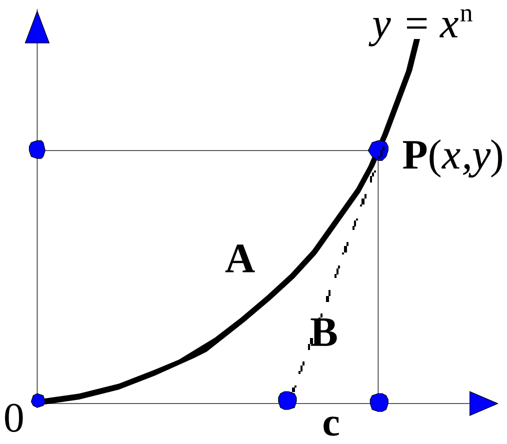
Another look at the calculus power rules

by Sidney Schuman (Published in The Mathematical Gazette, July 2005)



At point P on the graph of $y = x^n \ (x > 0)$, a rectangle is formed by lines perpendicular to the coordinate axes and a right triangle is formed by a line tangent to the curve. The rectangle is divided by the curve into regions A and B, the triangle has base c. A previous article of mine (Tinkering with the calculus power rules) used two diagrams to support the algebra that showed how, on the one hand, $\frac{\mathrm{A}}{\mathrm{B}} = n$ is obtained from the integral power rule and, on the other hand, $\frac{x}{\mathrm{c}} = n$ is obtained from the differential power rule. These were treated seperately with no attempt to show a connection between them.

This connection is, of course, a heavily-disguised version of the inverse relationship between the calculus power rules. This relationship is shown easily enough with a standard piece of algebra, but this begs an interesting, albeit naive, question. '*Why* is an area rule the inverse of a gradient rule?' It occurred to me that combining these diagrams into a single diagram (above) might provide a way of addressing that question.

This seems to suggest that, as point P moves along the curve, the tangent line 'projects' the ratio $\frac{\mathrm{A}}{\mathrm{B}}$ onto the x-axis. In other words, just as region B is always $\frac{1}{n}$th of region A, so the base of the triangle is always $\frac{1}{n}$th of the base of the rectangle. Since these ratios are merely 'boiled-down' versions of the calculus power rules, this could be a way of showing graphically the reason why they are connected.

I wonder if it is possible to devise a mathematical formulation, based on this 'dynamic' geometry, that answers my naive question simply and convincingly?