Before teaching calculus

by Sidney Schuman (published in Mathematics Teaching, March 2001)

In most teaching of calculus to sixth-form students, there is an assumption that they understand the concept of gradient. I want to suggest that perhaps this is not best practice. Most sixth-formers will, after all, have had only limited experience of gradient (performing friction experiments, drawing linear graphs, cycling up a hill) and the concept of gradient may not have been internalised securely from their experience. It is no surprise then that, after happily calculating the point at which a secant becomes a tangent, their attempt to understand the underlying theory quickly become almost theological in character, with ‘gradient of a curve’ and ‘rate-of-change’ being the main dogma.

This problem arises because the cardinal rule of maths teaching has been overlooked, ie that learning maths is like building a house - it’s no good trying to put the roof on until the walls are in place, and the walls cannot be constructed without a proper foundation. This problem can be avoided if calculus is introduced not in terms of gradient, which may not be understood, but in terms of area, which almost certainly will be.

We learn about area soon after we learn about number. Our understanding of area is reinforced continually by our experiences (material to make a dress, carpet to cover a floor, paint to cover a surface, etc) so that the concept becomes successfully internalised. Compared to the somewhat exotic concerns of differential calculus (eg: rates of change) the business of integral calculus (eg: sums) relates directly to everyday experience. This suggests that area may be a better introduction to calculus than gradient and that the integral, rather than the differential, power rule should be the point of departure. All that is needed is a suitable method of obtaining the integral power rule independently, rather than merely as an inverse. By ‘suitable’ I mean one that is comparable to the numerical method commonly used to show how a secant transforms to a tangent.

Such a method does exist and I suggest, with all due modesty, that it makes it possible for students to feel good about their subsequent calculus studies. In **Power Maths** both calculus power rules are obtained (without reference to the concept of the limit or infinitesimals) in a numerical graph-based investigation. Since they are obtained independently, students are also able to find the inverse relationship themselves. The advantage of using such a pre-calculus ‘taster’ is that students can gain confidence in the provenance and use of the two calculus power rules, enabling them to more easily accept the underlying theory later on. In other words, with their area *foundation* we can help them build their calculus power rule *walls* and put the calculus theory *roof* on later.

This ‘hands-on’ approach can no more guarantee that concepts will eventually be learned than do any of the current methods. What it may do, however, is to go some way towards removing the fear of calculus among sixth-form students - and surely this is a worth-while aim.