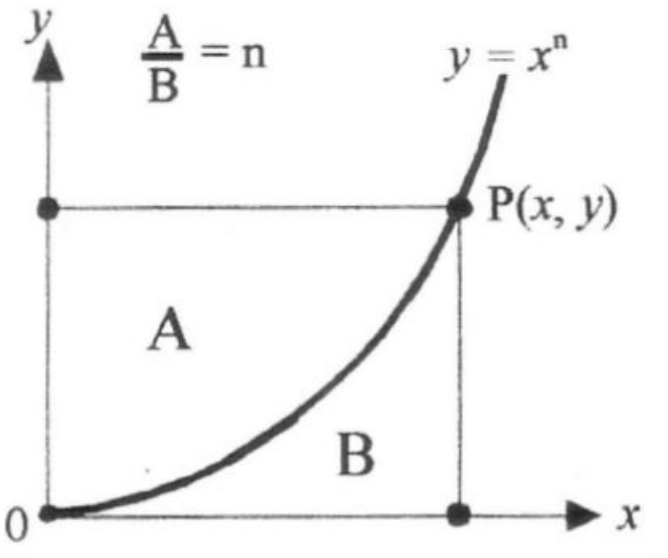
Power Maths by Sidney Schuman

Power Maths is the name given to a graph-based non-rigorous geometric method of teaching the calculus power rules. Students using this method can discover the rules without making any gradient calculations and without first having to understand the concept of the limit. The method is based on the curious property that each calculus power rule can be reduced to a simple geometric ratio. The integral pwer rule can be reduced to a ratio of areas; the differential power rule can be reduced to a ratio of distances. These ratios are easy for the student to establish by investigation, leaving only some elementary algebra to make the connection from ratio to rule, as descibed below.



To find the integral power rule, sketch graphs of $y = x^2, \quad y = x^3, \quad y = x^4$ etc will be needed, drawn for $0 < x < 10$.

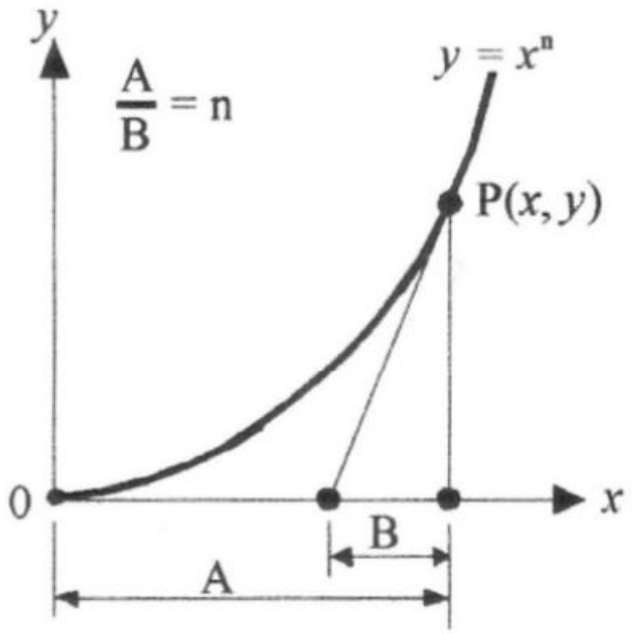
Using the mid-ordinate rule with 10 strips, the approximmate area of region B can be calculated. The rectangle area is xy and hence area $A = xy - B$. Confirm that A / B = n with a very small error (which could be made smaller by using a greater nimber of strips).

The rule is then derived as follows:

$$\frac{A}{B} = n \quad \therefore A = Bn; \quad A + B = xy \quad \therefore A + B = x^{n+1}$$

$$Bn + B = x^{n+1} \quad \therefore B(n+1) = x^{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$$

Thus we have arrived at the integral power rule, normally written as: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.



To find the differential power rule, sketch graphs of $y = x^2, \quad y = x^3, \quad y = x^4$ etc will be needed, drawn for $0 < x < 10$.

Draw a tangent line at a specific point (P) and measure the distances A and B. Confirm that A/ B = n by using several points on each graph and repeating this routine.

The rule is then derived as follows:

$$\frac{A}{B} = n \quad \text{but } A = x \quad \therefore \frac{x}{B} = n \quad \therefore B = \frac{x}{n}$$

Gradient of tangent $= \frac{y}{B} = \frac{x^n}{B} = \frac{x^n}{x/n} = nx^{n-1}$.

Thus we have arrived at the differential power rule, normally written as: $\frac{dy}{dx} = nx^{n-1}$.

Practice in using each rule on simple monomial functions can be followed by self-checking exercises using both rules, enabling the student to discover their inverse relationship. The student should now be more amenable to those aspects of calculis (infinitesimals, concept of the limit, formal notation) that cause the most anxiety.

This is an edited version of an article that appeared in Mathematics Teaching, March 2005.