Tinkering with the calculus power rules

by Sidney Schuman (published in The Mathematical Gazette, July 2003)

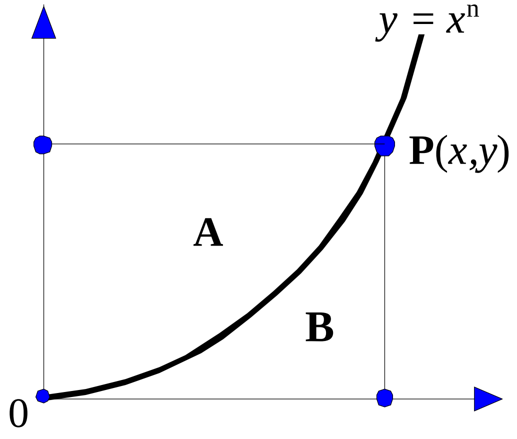

Figure 1

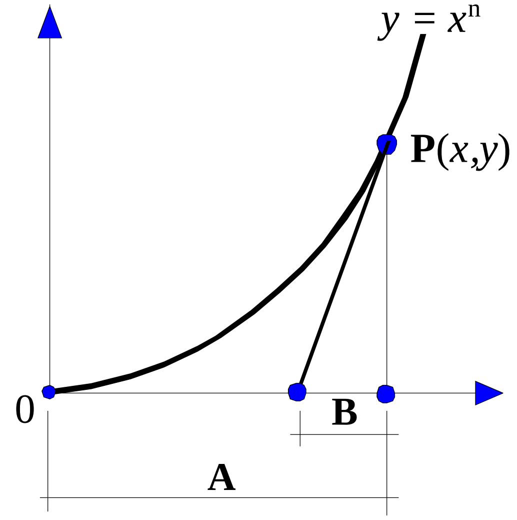

Figure 2

The graph of the power function $y = x^n,\ x > 0, n > 0$ yields an intriguing piece of geometry in relation to the calculus power rules. Given the simplest form of the differential and integral power rules (omitting any coefficients and constants), it can be shown that the power 'n' is equal to $\frac{\text{A}}{\text{B}}$, where A and B are significant areas (Figure 1) or significant dimensions (Figure 2).

Figure 1: Let A be the area of the region above and B the region below the curve $y = x^n$, these regions being contained within the rectangle formed by lines drawn perpendicular to the coordinate axes from point *P*(*x*,*y*) on the curve. The area of this rectangle is the product of the coordinates at *P* and since $y = x^n$, this area is equal to x^{n+1}. Given that by the integral power rule $\text{B} = \int x^n dx = \frac{x^{n+1}}{n+1}$ (disregarding the constant of integration), we note that $\text{A} = x^{n+1} - \frac{x^{n+1}}{n+1} \quad \therefore \text{A} = n\left(\frac{x^{n+1}}{n+1}\right)$. Thus it follows that: $\frac{\text{A}}{\text{B}} = n$.

Figure 2: Let A be the horizontal displacement *x* of point P(*x*,*y*) on the curve of $y = x^n$ and B the base length of a right triangle with apex at P(*x*,*y*) and hypotenuse tangent to the curve. We note that at P(*x*,*y*), the gradient of the curve is the same as the slope of the hypotenuse which by simple geometry is equal to $\frac{x^n}{\text{B}}$.

Given that the differential power rule $\frac{dy}{dx} = nx^{n-1}$ describes the gradient of the curve at any point on the curve, then at P(*x*,*y*) we have: $nx^{n-1} = \frac{x^n}{\text{B}} \quad \therefore \frac{x}{\text{B}} = n \quad \therefore \frac{\text{A}}{\text{B}} = n$