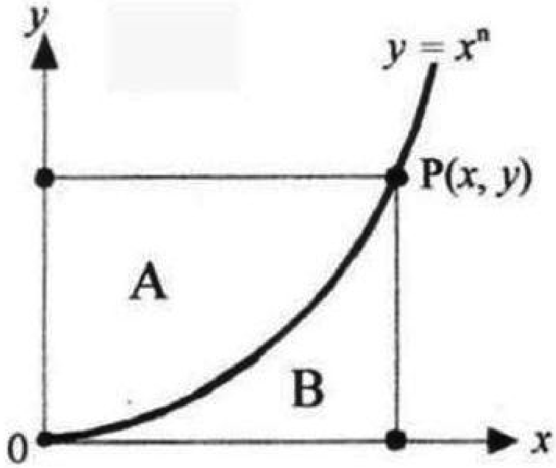
Tinkering with the calculus power rules

by Sidney Schuman

The graph of the power function $y = x^{\mathrm{n}}$, $x > 0$, $\mathrm{n} > 0$ yields an intriguing piece of geometry in relation to the calculus power rules. Given the simplest form of the rules (omitting any coefficients and constants) it can be shown that the power 'n' is equal to A / B where A and B are significant areas or significant distances.

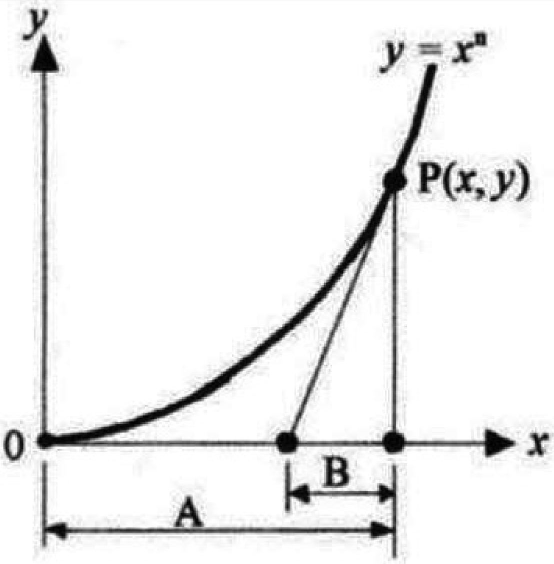


Let area A be the area of the region above and area B the area of the region below the curve $y = x^{\mathrm{n}}$, these regions being contained within the rectangle formed by lines drawn perpendicular to the coordinate axes from point P(x,y) on the curve. The area of this rectangle is the product of the coordinates at P(x,y) and, since $y = x^{\mathrm{n}}$, this area is $x^{\mathrm{n+1}}$.

Given that by the integral power rule B = $\int x^{\mathrm{n}} dx = \frac{x^{\mathrm{n+1}}}{\mathrm{n+1}}$,

we note that since $\frac{x^{\mathrm{n+1}}}{\mathrm{n+1}} + \mathrm{n}\frac{x^{\mathrm{n+1}}}{\mathrm{n+1}} = x^{\mathrm{n+1}}$, then A = $\mathrm{n}\frac{x^{\mathrm{n+1}}}{\mathrm{n+1}}$.

Therefore $\frac{\mathbf{A}}{\mathbf{B}} = \mathbf{n}$



Let A be the distance horizontally from the origin to P(x,y) and B the base length of the right triangle formed by tangent and ordinate from P(x,y) to the x-axis.

Given that by the differential power rule the gradient of the curve is $\frac{dy}{dx} = \mathrm{n}x^{\mathrm{n}-1}$, we note that since the gradient of the tangent is $\frac{x^{\mathrm{n}}}{\mathrm{B}}$, then $\mathrm{n}x^{\mathrm{n}-1} = \frac{x^{\mathrm{n}}}{\mathrm{B}}$ $\therefore \mathrm{B} = \frac{x^{\mathrm{n}}}{\mathrm{n}x^{\mathrm{n}-1}} = \frac{x}{\mathrm{n}}$ $\therefore \mathrm{n} = \frac{x}{\mathrm{B}}$.

But by definition, $x = \mathrm{A}$, so therefore $\frac{\mathbf{A}}{\mathbf{B}} = \mathbf{n}$.

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