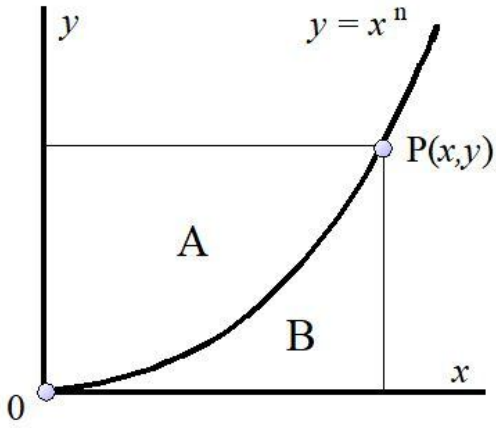
Power Maths Worksheet by Sidney Schuman

Go straight to the integral power rule

On the graph of $y = x^n$ a rectangle has been drawn from point $P(x, y)$ on the curve. It is divided by the curve into two regions.



It can be shown algebraically that $\frac{A}{B} = n$.

To confirm this numerically,
Let $x = 10$ and divide the rectangle into 10 equal-width vertical strips.
Calculate area B for values of **n = 2, 3, 4, 5** using the mid-ordinate rule:

$B = 0.5^n + 1.5^n + 2.5^n + 3.5^n + 4.5^n + 5.5^n + 6.5^n + 7.5^n + 8.5^n + 9.5^n$

The rectangle area for each value of n is equal to 10^{n+1}
Calculate area A for each value of **n** using the formula: $A = 10^{n+1} - B$

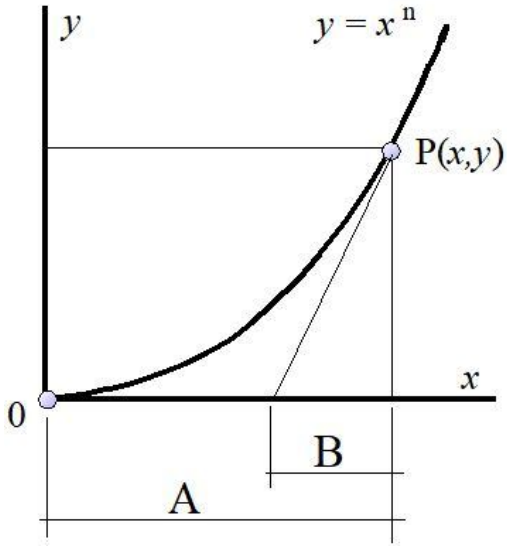
Find the ratio A/B for each value of n. Your calculations should indicate that $\frac{A}{B} = n$ ***Equation 1***

Now, rectangle area for any value of x is x^{n+1} so it follows that $A + B = x^{n+1}$ ***Equation 2***

Combine Equations 1 and 2 and deduce the integral power rule.

Go straight to the differential power rule

On the graph of $y = x^n$ another line has been drawn at $P(x, y)$ tangent to the curve. With the ordinate this forms a right triangle with base B.



It can be shown algebraically that $\frac{A}{B} = n$.

To confirm this practically, you will need pre-drawn graphs of $y = x^n$
These are available for values of **n = 2, 3, 4, 5**.
Use the link on the home page to download and print these graphs.
Draw tangent and ordinate from a point approx. midway on each graph.

Check dimensions A and B and confirm that $\frac{A}{B} = n$. ***Equation 3***

Equal gradients at $P(x, y)$ means $\frac{dy}{dx} = \frac{y}{B} \quad \therefore \frac{dy}{dx} = \frac{x^n}{B}$ ***Equation 4***

Combine Equations 3 and 4 and deduce the differential power rule.