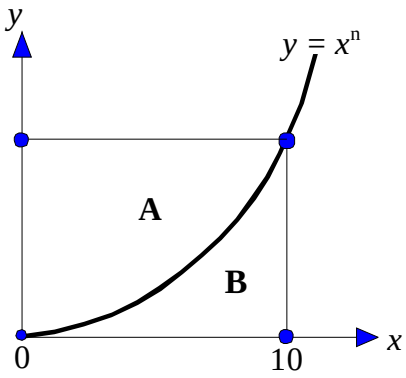
Power Maths by Sidney Schuman

Use this worksheet to go straight to the calculus power rules

---

Integral power rule



On the graph of $y = x^n$ a rectangle has been drawn from the point on the curve where $x = 10$. This rectangle is divided by the curve into two regions, A and B. Let ten vertical strips of equal width fill the rectangle. The area 'below' the curve (B) can now be calculated using the mid-ordinate rule. Because the vertical strips are one unit wide, the approximate area of each vertical strip is simply the height of the mid-ordinate,

so $\mathrm{B} = 0.5^n + 1.5^n + 2.5^n + 3.5^n + 4.5^n + 5.5^n + 6.5^n + 7.5^n + 8.5^n + 9.5^n$

This can then be subtracted from the rectangle area to give the area 'above' the curve (A).

Calculate the area of regions A and B for n = 2, 3, 4 and 5; find the ratio of areas $\frac{\mathrm{A}}{\mathrm{B}}$ for each value of n

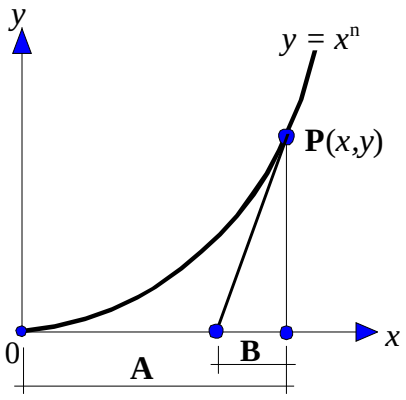
Your calculations should indicate that: $\frac{\mathrm{A}}{\mathrm{B}} = n$ *Equation 1*

Now, the area of the rectangle is xy but $y = x^n$ so the rectangle area is x^{n+1}

But the rectangle area is also the sum of the region areas A and B, so: $\mathrm{A} + \mathrm{B} = x^{n+1}$ *Equation 2*

Combine Equation 1 and Equation 2 to deduce the integral power rule.

Differential power rule



On the graph of $y = x^n$ two lines have been drawn from point $\mathrm{P}(x, y)$ on the curve. One is perpendicular to the *x*-axis and the other is tangent to the curve, forming a right triangle. As the point moves round the curve, the tangent line varies in length and orientation. The ratio of areas (above) is projected by the tangent line on to the *x*-axis and becomes a ratio of dimensions.

So again we have: $\frac{\mathrm{A}}{\mathrm{B}} = n$

Using the link, download and print the graphs of $y = x^2$, $y = x^3$, $y = x^4$ and $y = x^5$. Draw a tangent line and an ordinate from a point you have chosen on each graph. Measure the dimensions A and B and confirm that their ratio is equal to the power of x.

Now, given that $\mathrm{A} = x$ and $\frac{\mathrm{A}}{\mathrm{B}} = n$, we have: $\frac{x}{\mathrm{B}} = n$ *Equation 3*

And the rate of change $\frac{dy}{dx}$ of the curve $y = x^n$ is also the gradient (m) of the hypotenuse

But, $\mathrm{m} = \frac{y}{\mathrm{B}}$ and since $y = x^n$ then $\mathrm{m} = \frac{x^n}{\mathrm{B}}$ *Equation 4*

Combine Equation 3 and Equation 4 to deduce the differential power rule.
