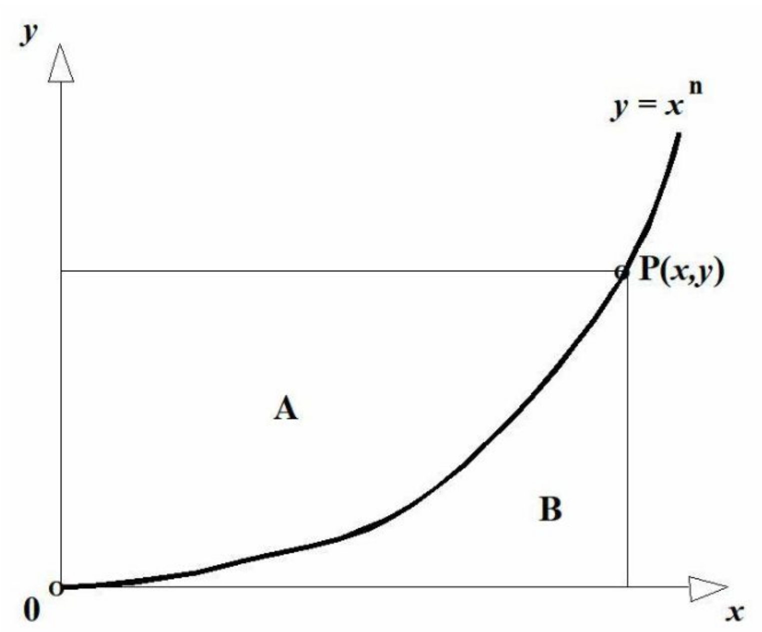
Reducing the calculus power rules

By Sidney Schuman

The integral power rule



$\int x^n dx = \frac{x^{n+1}}{n+1}$ gives the area 'below the curve' of

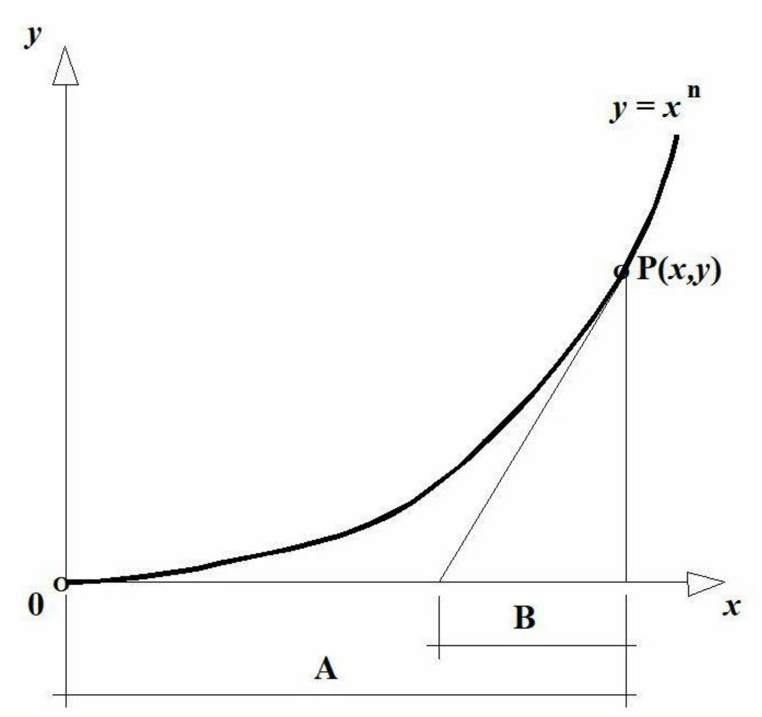
$y = x^n$ for $0 < x < \text{P}$, $\therefore B = \frac{x^{n+1}}{n+1}$ $\therefore B = \frac{xy}{n+1}$ (1)

For the rectangle area drawn at P, $A + B = xy$

$\therefore A = xy - B$ $\therefore A = xy - \frac{xy}{n+1}$ $\therefore A = n\frac{xy}{n+1}$ (2)

Comparing (1) and (2) we can see that $\mathbf{\frac{A}{B} = n}$

The differential power rule



$\frac{dy}{dx} = n\,x^{n-1}$ gives the value of gradient at any point on the curve of $y = x^n$. At P a tangent can be seen as the hypotenuse of a right triangle drawn down to the *x*-axis.

The gradient of the hypotenuse is $m = \frac{y}{B}$ $\therefore m = \frac{x^n}{B}$.

This is equal to the gradient of the curve at P, so we have
$\frac{x^n}{B} = n\,x^{n-1}$ $\therefore B = \frac{x^n}{n\,x^{n-1}}$ $\therefore B = \frac{x}{n}$.

But $x = A$ $\therefore B = \frac{A}{n}$ $\therefore \mathbf{\frac{A}{B} = n}$

This is an edited version of the article 'Tinkering with the calculus power rules' published in The Mathematical Gazette, July 2003