A new look at the integral power rule

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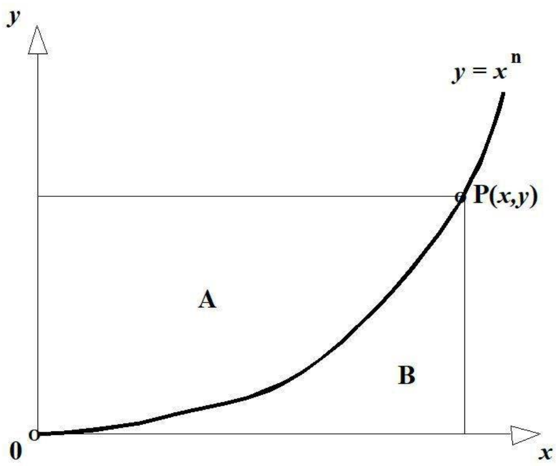


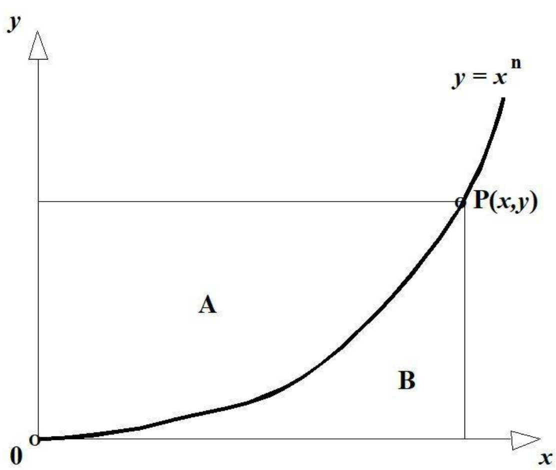
$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{so in the diagram} \quad B = \frac{x^{n+1}}{n+1} \qquad \text{(Equation 1)}$$

$$\text{Rectangle area } = xy = x^{n+1} \quad \therefore A + B = x^{n+1} \quad \therefore A = x^{n+1} - B$$

$$\therefore A = x^{n+1} - \frac{x^{n+1}}{n+1} \quad \therefore (n+1)A = (n+1)x^{n+1} - x^{n+1} = n\, x^{n+1}$$

$$\therefore A = n\frac{x^{n+1}}{n+1} \text{ and comparing this to Eq 1 we see that } \frac{A}{B} = n$$

Bearing in mind that most students will understand area more easily than gradient, the ratio A/B = n can be used to introduce the integral power rule as a pre-calculus taster. So, let's go **straight to the integral power rule**.



The rectangle area is width times height = $xy = x^{n+1}$ (since $y = x^n$)

This gives us by observation our first equation: $A + B = x^{n+1}$ (Eq 1)

Calculate area **B** for values of x and **n** using the **mid-ordinate rule**

Since **A**= rectangle area **- B**, confirm that $\frac{A}{B} = n \quad \therefore A = Bn$ (Eq 2)

Substituting in Eq 1 $\quad Bn + B = x^{n+1} \quad \therefore B(n+1) = x^{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$

This is the integral power rule, $\int x^n dx = \frac{x^{n+1}}{n+1}$ (constant of integration omittrd)

Using the mid-ordinate rule with (say) ten strips for x =10 and various values of n gives students the opportunity of finding the integral rule as a simple calculator exercise, with no reference to calculus theory.

Reduction of the integral power rule was published in TheMathematical Gazette in July 2003 as 'Tinkering with the calculus power rules'.